Write your name here

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**Pearson Edexcel GCE**

Centre Number | Candidate Number

# AS and A level Further Mathematics
# Further Statistics 2

# Practice Paper
# Continuous probability distributions

**You must have:**
Mathematical Formulae and Statistical Tables (Pink)

Total Marks

**Instructions**

- Use black ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- Fill in the boxes at the top of this page with your name, centre number and candidate number.
- Answer all the questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – there may be more space than you need.
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

**Information**

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 10 questions in this question paper. The total mark for this paper is 101.
- The marks for each question are shown in brackets – use this as a guide as to how much time to spend on each question.
- Calculators must not be used for questions marked with a * sign.

**Advice**

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

**1.** The time in minutes that Elaine takes to checkout at her local supermarket follows a continuous uniform distribution defined over the interval [3, 9].

Find

(*a*) Elaine's expected checkout time,

**(1)**

(*b*) the variance of the time taken to checkout at the supermarket,

**(2)**

(*c*) the probability that Elaine will take more than 7 minutes to checkout.

**(2)**

Given that Elaine has already spent 4 minutes at the checkout,

(*d*) find the probability that she will take a total of less than 6 minutes to checkout.

**(3)**

**(Total 8 marks)**

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**2.** The random variable $X$ has a continuous uniform distribution on $[a, b]$ where $a$ and $b$ are positive numbers.

Given that $\mathrm{E}(X) = 23$ and $\mathrm{Var}(X) = 75$,

(*a*) find the value of $a$ and the value of $b$.

**(6)**

Given that $\mathrm{P}(X > c) = 0.32$,

(*b*) find $\mathrm{P}(23 < X < c)$.

**(2)**

**(Total 8 marks)**

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**3.** The random variable $R$ has a continuous uniform distribution over the interval [5, 9].

(*a*) Specify fully the probability density function of $R$.

**(1)**

(*b*) Find $P(7 < R < 10)$.

**(1)**

The random variable $A$ is the area of a circle radius $R$ cm.

(*c*) Find $E(A)$.

**(4)**

**(Total 6 marks)**

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**4.** The continuous random variable $X$ is uniformly distributed over the interval $[\alpha, \beta]$

Given that $E(X) = 3.5$ and $P(X > 5) = \dfrac{2}{5}$

(*a*) find the value of $\alpha$ and the value of $\beta$

**(4)**

Given that $P(X < c) = \dfrac{2}{3}$

(*b*) (i) find the value of $c$

(ii) find $P(c < X < 9)$

**(3)**

A rectangle has a perimeter of 200 cm. The length, $S$ cm, of one side of this rectangle is uniformly distributed between 30 cm and 80 cm.

(*c*) Find the probability that the length of the shorter side of the rectangle is less than 45 cm.

**(4)**

**(Total 11 marks)**

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**5.** In a game, players select sticks at random from a box containing a large number of sticks of different lengths. The length, in cm, of a randomly chosen stick has a continuous uniform distribution over the interval [7, 10].

A stick is selected at random from the box.

(*a*) Find the probability that the stick is shorter than 9.5 cm.

**(2)**

To win a bag of sweets, a player must select 3 sticks and wins if the length of the longest stick is more than 9.5 cm.

(*b*) Find the probability of winning a bag of sweets.

**(2)**

To win a soft toy, a player must select 6 sticks and wins the toy if more than four of the sticks are shorter than 7.6 cm.

(*c*) Find the probability of winning a soft toy.

**(4)**

**(Total 8 marks)**

**6.** The continuous random variable $L$ represents the error, in metres, made when a machine cuts poles to a target length. The distribution of $L$ is a continuous uniform distribution over the interval [0, 0.5].

(*a*) Find $\mathrm{P}(L < 0.4)$.

**(1)**

(*b*) Write down $\mathrm{E}(L)$.

**(1)**

(*c*) Calculate $\mathrm{Var}(L)$.

**(2)**

A random sample of 30 poles cut by this machine is taken.

(*d*) Find the probability that fewer than 4 poles have an error of more than 0.4 metres from the target length.

**(3)**

When a new machine cuts poles to a target length, the error, $X$ metres, is modelled by the cumulative distribution function $\mathrm{F}(x)$ where

$$\mathrm{F}(x) = \begin{cases} 0 & x < 0 \\ 4x - 4x^2 & 0 \leq x \leq 0.5 \\ 1 & \text{otherwise} \end{cases}$$

(*e*) Using this model, find $\mathrm{P}(X > 0.4)$.

**(2)**

A random sample of 100 poles cut by this new machine is taken.

(*f*) Using a suitable approximation, find the probability that at least 8 of these poles have an error of more than 0.4 metres.

**(3)**

**(Total 12 marks)**

**7.** A continuous random variable $X$ is uniformly distributed over the interval $[b, 4b]$ where $b$ is a constant.

(*a*) Write down $\mathrm{E}(X)$.

**(1)**

(*b*) Use integration to show that $\mathrm{Var}(X) = \frac{3b^2}{4}$.

**(3)**

(*c*) Find $\mathrm{Var}(3 - 2X)$.

**(2)**

Given that $b = 1$, find

(*d*) the cumulative distribution function of $X$, $\mathrm{F}(x)$, for all values of $x$,

**(2)**

(*e*) the median of $X$.

**(1)**

**(Total 9 marks)**

**8.** The continuous random variable $X$ is uniformly distributed over the interval $[-4, 6]$.

(*a*) Write down the mean of $X$.

**(1)**

(*b*) Find $P(X \leq 2.4)$.

**(2)**

(*c*) Find $P(-3 < X - 5 < 3)$.

**(2)**

The continuous random variable $Y$ is uniformly distributed over the interval $[a, 4a]$.

(*d*) Use integration to show that $E(Y^2) = 7a^2$.

**(4)**

(*e*) Find $\text{Var}(Y)$.

**(2)**

(*f*) Given that $P(X < \frac{8}{3}) = P(Y < \frac{8}{3})$, find the value of $a$.

**(3)**

**(Total 14 marks)**

**9.** A piece of string $AB$ has length 9 cm. The string is cut at random at a point $P$ and the random variable $X$ represents the length of the piece of string $AP$.

(*a*) Write down the distribution of $X$.

**(1)**

(*b*) Find the probability that the length of the piece of string $AP$ is more than 6 cm.

**(1)**

The two pieces of string $AP$ and $PB$ are used to form two sides of a rectangle.

The random variable $R$ represents the area of the rectangle.

(*c*) Show that $R = aX^2 + bX$ and state the values of the constants $a$ and $b$.

**(2)**

(*d*) Find $\mathrm{E}(R)$.

**(6)**

(*e*) Find the probability that $R$ is more than twice the area of a square whose side has the length of the piece of string $AP$.

**(4)**

**(Total 14 marks)**

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**10.** The continuous random variable $X$ is uniformly distributed over the interval $[-1, 3]$.

Find

($a$) $E(X)$

**(1)**

($b$) $Var(X)$

**(2)**

($c$) $E(X^2)$

**(2)**

($d$) $P(X < 1.4)$

**(1)**

A total of 40 observations of $X$ are made.

($e$) Find the probability that at least 10 of these observations are negative.

**(5)**

**(Total 11 marks)**

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**TOTAL FOR PAPER: 101 MARKS**